Adding Momentum and Reversion

To Multi-Factor Asset Pricing Models

Kevin C. Kaufhold

2008:2

February, 2008

Abstract

In this paper, momentum and reversionary factors are placed into the multi-factor model initially developed by Fama and French (1996). Using updated data and developing prior returns for the dependent asset portfolios, momentum and short-term reversion factors are found to not be adequately modeled in the three-factor model, as the alpha intercept is far above zero. Fama and French also found that the three-factor model failed to adequately explain momentum over a 2 to 12 month period. The situation improves with a long-term reversionary factor, however. The paper also explores the addition of a fourth independent factor that more directly assesses momentum and reversion to the mean. The four-factor process improves upon all results based upon prior returns. Short-term reversion and momentum both can be better explained with four independent variables, and even long-term reversion has some modest improvements in results. Momentum was also found to be better modeled with a four-factor model in Carhart 91997), and this paper builds upon that work with updated data.

Table of Contents

1. Introduction

Prior research (Kaufhold, 2008:1) explored the addition of Fama and French size and value factors to the Capital Asset Pricing Model (CAPM). Using updated data for time period of 1963 to 2006 (522 months), it was shown that the 3-factor model of Fama and French (1996) provided significantly greater explanatory power in asset pricing variation then through just the CAPM. This was true regardless of how the dependent asset portfolios were formed, with size-price to book, earnings to price, dividend yield, cash flow to price, price to book (only), and size (only) all being used for the individual asset portfolios. The market risk premium, size (SMB), and value (HML) factors were individually and jointly significant, as well. The coefficients on the independent variables were positive, indicating a direct relationship with asset pricing and the three factors. Further, the y intercept, alpha, was much closer to 0 with the three-factor model than with the CAPM. The hypothesis that H_0: $\alpha_i = 0$, is more difficult to reject with the multi-factor pricing model than with merely the CAPM.

Thus, the capital markets may be conveying useful information through size and value considerations. Impliedly, active management utilizing size and value style may be able to capture equity premiums. There is still the open question of whether such premiums can exceed the cost of research and tax impacts, but at least on a pre-tax, pre-cost basis, there may value to the value style of investing. The positive findings as to the size factor may be less telling, since research suggests that a micro-cap effect may be largely driving this factor. See, O'Shaughnessy (1998); Fama and French (2007).

Additionally, no consensus exists as to the underlying "state" variables that may be involved in a size or value premium. Fama and French (1995) believe that financial stress of value-oriented firms may be responsible. With this argument, the higher risk of firms in distress justifies the higher return of value equities. Thus, the markets may still be considered rational and efficient. Conversely, Chan and Lakonishok (2004) believe that market inefficiencies are involved, since value equities exhibit lower semi-variance and downside pricing volatility risk. The theoretical debate between efficiency and behavior has therefore shifted from whether there is value to value, to what explains the value.

This working paper extends the prior research conducted in 2008:1 by analyzing the role of momentum and reversionary factors in asset pricing. Part II examines the Fama and French method of creating dependent asset portfolios based on prior returns. Part III explores a four-factor model initially developed by Carhart (1997) that creates a momentum or reversionary factor as a fourth independent variable to se with dependent portfolios based on prior returns.

2. The Three-Factor FF Model

Literature Search on Past Returns. Studies have shown a momentum effect in the short-term. Jegadeesh and Titman (1993) found that stocks exhibit momentum of 3 to 12 month duration, with good or bad pricing performance continuing in the short-term. There is also some evidence (O'Shaughnessy, 1998, at 234-235) that one-year pricing performance (also known as relative strength) outperforms, but the volatility risk is extraordinarily high.

While there may be mean acceleration in short-term time frames, overwhelming evidence exists on mean reversion in the longer time periods. Fama and French (1988) found significant serial correlation with five-year return data, and the effect was more pronounced with the small cap firms. DeBondt and Thaler (1985, 1987) demonstrated that reversion to the mean was being experienced at the individual stock level, and not just in the aggregate markets. Poterba and Summers (1988) also found negative long-term serial correlation, as did Barberis (2000). Chopra, Lakonishok, and Ritter (1992) discovered that poorly performing stocks exhibited strong reversals in future time frames, leading to significant out-performance. Reichenstein & Dorsett (1995) showed that bad periods of market behavior were *predictably* followed by good periods, and visa versa. Siegel (1998, at 12, 33) even attributed the stability of real returns on stocks over extraordinarily long periods (of 195 years) to mean reversion tendencies in equities.

Substantial negative correlation exists in longer-term price behavior, in apparent contradiction to market efficiency (Damodaran, 1996, at 164). The degree of serial correlation in equity returns is so strong that profit opportunities may be possible, while other anomalies (such as the calendar effect) are usually not considered to be sufficient in their correlations to consistently allow abnormal profits. (Bodie, Kane, and Marcus, 2004, at 656).

Data and Methodology. Data sources for this study are ultimately based on CRSP and COMPUSTAT data files of the major domestic equity exchanges. The actual data source used was the data library of Prof. Kenneth French, freely available on-line to all researchers. Prof. French used the above referenced equity data files to structure equity returns into various asset portfolios and variables for use in regressions.

The central research question for this paper is whether a theoretical asset-pricing model can explain the momentum and reversionary effects in asset pricing. A series of regressions are generated from cross-sectional return data of all domestic equities on the major exchanges between July, 1963 and December, 2006. Since time intervals are involved, time series regressions are employed in the effort. Serial correlation between the independent variables and the residual error term is always a concern with time series work. Fortunately, any such serial correlation can be corrected through one of several different methods, making inferential testing valid. The Prais-Winston procedure was used in the regression runs, with the alternate Durbin-Watson statistic being reported, to correct for serial correlation in the estimates.

Fama and French (1993, 1996) developed a multi-factor asset pricing model that expanded upon the single-factor market model of the CAPM. The model can be stated as:

$$E(R_{it}) - R_{ft} = \alpha_i + b_i\,(E(R_{mt}) - R_{ft}) + s_i\,E(SMB_t) + h_i\,E(HML_t) + \varepsilon_{it} \qquad (1)$$

Where, $E(R_i)$ is the expected return on asset i; R_f is the risk-free rate of return; the difference of $E(R_i) - R_f$ is the asset risk premium that must be offered to induce investors to assume asset pricing volatility risk; alpha, αi, is the y intercept; R_m is the return of the market; and ε_i is the residual error term. α_i should be zero in a mean-variance efficient market. To expand on this further, if all independent variables fully capture or explain the return structure to asset pricing, then the alpha y intercept should not explain any of the pricing, thereby generating a zero return for the intercept. Both the CAPM and multi-factor models essentially become a hypothesis test of H_0: $\alpha_i = 0$.

SMB is the size variable, being composed of a set of mimicking portfolios that measure the return on small cap equities minus the return on large cap equities; and HML is the value variable, being a set of mimicking portfolios measuring the return on high book-to-market equities minus the return of low book-to-market equities. $E(R_{mt}) - R_{ft}$, E (SMB_t), and E (HML_t) are expected premiums of the market, size, and value. b_i , s_i , and h_i are factor sensitivities of the premiums, being the slopes or beta coefficients for each factor of a regression equation. See, Fama and French (1996), at 56.

For purposes of this study, E(Ri) is composed of monthly portfolio returns that are equally-weighted and divided into deciles. Portfolios are preferred for the dependent variable rather than individual assets since betas of diversified portfolios are more precise. See, Black, Jensen, and Scholes (1972); and Fama and French (1993). Since the focus of this study is to identify the effect that momentum or reversion has on asset returns, the dependent portfolios are now based upon prior returns. This is distinct from asset portfolios being sorted by size and value to price, as was the case in 2008:1. No size or value indicators exist in these dependent portfolios, as we are only interested in the possibility that past returns are either positively or negatively correlated with current returns. The portfolios are set for prior one-month returns (Pr1); prior 12 to 2 month returns (Pr12); and prior 60 to 13 month returns (Pr60).

Using prior return portfolios rather than portfolios sorted by size and price-to-book has the additional advantage of reducing the possibility of spurious results being generated because of size and value parameters being measured on both sides of the regression equation. This possibility seems to be remote, since research has shown that having different dependent portfolio compositions did not affect results (2008:1). But, at least the variables are now simply testing returns, without having to worry over the type of sorting conducted on those returns.

Using prior return asset portfolios for the dependent variable follows similar efforts by Fama and French (1996), who found that a three-factor model based on prior return portfolios cannot adequately explain short-term mean acceleration, but does a reasonable job of explaining longer-term mean reversion of asset pricing.

Estimation Results. The following tables show the summary statistics of dependent asset portfolios formed on prior returns.

Table I - Dependent Variable, Equal weighted, Prior Returns 1 - 1 month

Port	Obs	Mean	Std Dev	t Value	p > t
Pr1Dec1	522	1.9955	8.5011	5.36	<.0001
Pr1Dec2	522	0.9914	6.2805	3.61	0.0003
Pr1Dec3	522	0.8146	5.6438	3.3	0.001
Pr1Dec4	522	0.8490	5.3005	3.66	0.0003
Pr1Dec5	522	0.7497	5.0703	3.38	0.0008
Pr1Dec6	522	0.7488	4.8742	3.51	0.0005
Pr1Dec7	522	0.6637	4.8212	3.15	0.0018
Pr1Dec8	522	0.6020	5.0008	2.75	0.0062
Pr1Dec9	522	0.4288	5.3253	1.84	0.0664
Pr1Dec10	522	-0.2067	6.4162	-0.74	0.4621

Table II - Dependent Variable, Equal weighted, Prior Returns 12 - 2 months

Port	Obs	Mean	Std Dev	t Value	p > t
Pr12Dec1	522	0.3747	8.7484	0.98	0.3283
Pr12Dec2	522	0.5805	6.1780	2.15	0.0323
Pr12Dec3	522	0.6879	5.4612	2.88	0.0042
Pr12Dec4	522	0.7246	4.9882	3.32	0.001
Pr12Dec5	522	0.8368	4.8484	3.94	<.0001
Pr12Dec6	522	0.8463	4.7327	4.09	<.0001
Pr12Dec7	522	0.9853	4.7960	4.69	<.0001
Pr12Dec8	522	1.0631	4.9497	4.91	<.0001
Pr12Dec9	522	1.2720	5.3647	5.42	<.0001
Pr12Dec10	522	1.5502	6.7387	5.26	<.0001

Table III - Dependent Variable, Equal weighted, Prior Returns 60-13 months

Port	Obs	Mean	Std Dev	t Value	p > t
Pr60Dec1	522	1.5011	7.8345	4.38	<.0001
Pr60Dec2	522	1.0346	5.8463	4.04	<.0001
Pr60Dec3	522	0.9063	5.2478	3.95	<.0001
Pr60Dec4	522	0.8688	4.9606	4	<.0001
Pr60Dec5	522	0.8939	4.6672	4.38	<.0001
Pr60Dec6	522	0.8639	4.6183	4.27	<.0001
Pr60Dec7	522	0.8184	4.6768	4	<.0001
Pr60Dec8	522	0.7614	4.8331	3.6	0.0003
Pr60Dec9	522	0.7036	5.2622	3.05	0.0024
Pr60Dec10	522	0.3844	6.3081	1.39	0.1644

The summary statistics show that portfolios formed on prior one-month returns (Table I) and prior 60 to 13 month returns (Table III) mean revert, with the deciles of lowest past returns produce far higher current returns than the deciles of the highest past returns. These statistics demonstrates both a very short-term reversal (of a single month) as well as a longer-term reversal of up to 60 months in duration. Table II shows a momentum effect, with mean acceleration occurring in portfolios formed using returns in the prior 12 to 2 months prior to portfolio formation.

The following table summarizes the regression outputs for prior 1 month dependent portfolios (short-term reversal), sorted by deciles for low return to high return. Output for both the CAPM and the three-factor model are reported.

Table IV - 1963-2006, Prior 1 month Dependent Portfolios (ST Rev)

CAPM ---- $R_i - R_f = a + b (R_m - R_f) + u_i$

	Pr1-1	Pr1-2	Pr1-3	Pr1-4	Pr1-5	Pr1-6	Pr1-7	Pr1-8	Pr1-9	Pr1-10
a	1.3105	0.4392	0.3126	0.3792	0.2991	0.3237	0.2452	0.1607	-0.0523	-0.7406
t(a)	5.13	2.55	2.08	2.63	2.13	2.30	1.70	1.09	-0.36	-0.37
b	1.4367	1.1582	1.0522	0.9928	0.9446	0.8904	0.8764	0.9233	1.0077	1.1191
t(b)	25.21	32.75	34.10	36.63	37.04	35.37	35.46	38.03	38.22	28.93
adj. R2	0.55	0.67	0.69	0.72	0.72	0.71	0.71	0.74	0.74	0.62
F test	635.33	1072.74	1162.64	1340.91	1371.65	1251.24	1257.4	1446.38	1460.49	836.83

Three-Factor Model ---- $R_i - R_f = a + b (R_m - R_f) + s (SML) + h (HML) + u_i$

	Pr1-1	Pr1-2	Pr1-3	Pr1-4	Pr1-5	Pr1-6	Pr1-7	Pr1-8	Pr1-9	Pr1-10
a	1.1056	0.1947	0.0574	0.0992	0.0327	0.0648	-0.0025	-0.0660	-0.2542	-0.9387
t(a)	5.37	1.63	0.56	1.17	0.35	0.76	-0.03	-0.66	-2.49	-5.44
b	1.2691	1.0765	0.9917	0.9663	0.9265	0.8695	0.8588	0.9020	0.9674	1.0520
t(b)	26.08	44.37	48.37	58.18	57.24	57.05	55.25	57.59	51.88	17.36
s	1.1864	0.8994	0.7913	0.7147	0.6509	0.6648	0.6416	0.6212	0.6513	0.2575
t(s)	18.81	28.21	29.34	32.65	30.27	32.88	31.03	29.67	26.37	4.87
h	0.2679	0.3278	0.3512	0.3758	0.3657	0.3507	0.3327	0.3016	0.2661	0.2575
t(h)	3.62	8.65	10.95	14.41	14.18	14.48	13.40	-0.66	9.00	4.87
adj. R2	0.73	0.87	0.88	0.91	0.90	0.91	0.90	0.90	0.89	0.76
F test	470.76	1162.90	1318.55	1797.75	1650.41	1720.50	1584.69	1652.32	1386.47	538.64

As is plainly evident, the three-factor model possesses superior explanatory power over the single factor market model. However, for the low and high deciles of the three-factor model, the intercept values are vastly different than zero. This significantly affects the average intercept value.

For the momentum dependent portfolios, the alpha intercepts have even greater values off of zero, as is seen in the below tables. This was also found in Fama and French (1996), who felt the three-factor model failed to adequately explain short-term mean acceleration.

Table V - 1963-2006, Prior 12 - 2 month Dependent Portfolios (Mom)

CAPM ---- $R_i - R_f = a + b (R_m - R_f) + u_i$

	Pr12-1	Pr12-2	Pr12-3	Pr12-4	Pr12-5	Pr12-6	Pr12-7	Pr12-8	Pr12-9	Pr12-10
a	-0.2764	0.0625	0.2145	0.2744	0.3999	0.4113	0.5442	0.6086	0.7883	0.9652
t(a)	-0.99	0.35	1.41	2.16	3.15	3.51	4.47	4.78	5.32	4.86
b	1.3655	1.0863	0.9923	0.9436	0.9158	0.9123	0.9241	0.9522	1.0121	1.2265
t(b)	21.34	28.05	31.06	35.14	35.96	38.26	39.38	39.52	36.38	31.85
adj. R2	0.47	0.60	0.65	0.70	0.71	0.74	0.75	0.75	0.72	0.66
F test	455.31	786.60	964.48	1234.82	1292.95	1463.43	1550.47	1561.43	1323.32	1014.47

Three-Factor Model ---- $R_i - R_f = a + b (R_m - R_f) + s (SML) + h (HML) + u_i$

	Pr12-1	Pr12-2	Pr12-3	Pr12-4	Pr12-5	Pr12-6	Pr12-7	Pr12-8	Pr12-9	Pr12-10
a	-0.5206	-0.2325	-0.1006	-0.0143	0.0960	0.1193	0.2506	0.3370	0.5159	0.7987
t(a)	-2.27	-1.91	-1.07	-0.18	1.33	1.78	3.31	3.68	4.44	4.70
b	1.1992	1.0222	0.9593	0.9195	0.9050	0.9016	0.9225	0.9449	1.0034	1.1617
t(b)	21.04	34.50	44.1	53.21	60.58	66.5	66.65	59.42	47.16	32.89
s	1.2388	0.8962	0.7927	0.6802	0.6651	0.6265	0.6003	0.5880	0.6170	0.7005
t(s)	16.86	23.40	28.05	30.19	33.92	35.14	32.74	27.82	21.90	15.12
h	0.3273	0.4048	0.4370	0.4047	0.4244	0.4106	0.4111	0.3784	0.3754	0.2122
t(h)	3.82	9.03	13.15	15.22	18.23	19.37	18.74	14.93	11.11	3.86
adj. R2	0.66	0.81	0.87	0.90	0.92	0.93	0.93	0.91	0.86	0.76
F test	332.03	750.82	1139.91	1535.24	1928.07	2248.38	2145.9	1675.39	1066.96	562.92

As to dependent asset portfolios based on long-term reversal between 60 and 13 month prior to portfolio formation, not only is the explanatory power of the three-factor model much higher than through the CAPM, but the y intercept values across all deciles are much closer to zero. Note also that the independent variables are statistically significant, both individually (t values) and jointly (F test for overall significance).

Table VI - 1963-2006, Prior 60 - 13 month Dependent Portfolios (LT Rev)

CAPM ---- $R_i - R_f = a + b (R_m - R_f) + u_i$

	Pr60-1	Pr60-2	Pr60-3	Pr60-4	Pr60-5	Pr60-6	Pr60-7	Pr60-8	Pr60-9	Pr60-10
a	0.9447	0.5334	0.4353	0.4118	0.4562	0.4251	0.3722	0.2974	0.1930	-0.2286
t(a)	3.32	31.30	3.17	3.34	4.06	4.00	3.55	2.76	1.73	-1.79
b	1.1657	1.0505	0.9871	0.9577	0.9180	0.9197	0.9351	0.9723	1.0703	1.2855
t(b)	20.22	30.09	34.04	37.63	41.21	43.18	44.79	46.94	48.45	46.45
adj. R2	0.44	0.63	0.69	0.73	0.77	0.78	0.79	0.81	0.82	0.81
F test	408.69	905.62	1158.96	1416.01	1697.87	1864.56	2005.66	2203.26	2347.68	2157.25

Three-Factor Model ---- $R_i - R_f = a + b (R_m - R_f) + s (SML) + h (HML) + u_i$

	Pr60-1	Pr60-2	Pr60-3	Pr60-4	Pr60-5	Pr60-6	Pr60-7	Pr60-8	Pr60-9	Pr60-10
a	0.4114	0.1404	0.0599	0.0729	0.1540	0.1542	0.1161	0.1050	0.0670	-0.1976
t(a)	1.91	1.30	0.71	1.04	2.28	2.18	1.68	1.34	0.71	-1.70
b	1.2414	1.0384	0.9956	0.9609	0.9304	0.9210	0.9363	0.9484	1.0186	1.1505
t(b)	20.62	43.85	52.21	66.39	68.84	71.64	68.08	68.33	61.38	50.15
s	0.7517	0.8470	0.6882	0.6540	0.5496	0.5357	0.5036	0.5186	0.5218	0.5800
t(s)	10.40	27.40	27.73	34.41	30.90	31.44	27.81	28.15	23.67	19.19
h	0.7417	0.5499	0.5334	0.4785	0.4259	0.3804	0.3605	0.2626	0.1646	-0.0627
t(h)	10.40	15.05	18.24	21.20	20.12	18.65	16.72	11.89	6.23	-1.74
adj. R2	0.70	0.86	0.89	0.93	0.93	0.93	0.92	0.93	0.91	0.89
F test	413.63	1081.10	1402.99	2207.04	2225.51	2376.12	2132.79	2189.44	1798.31	1343.22

Average absolute values and average squared values for alpha can be generated for all models and dependent portfolios, as is done in the following table. Results for both 1963 to 2006 and for 1963 to 1993 (from Fama and French, 1996) are included.

Table VII - Summary Results, CAPM and Three-Factor Models

						KCK 63-06			FF 63-93		
Sort	Weight		Explan.			ave abs a	ave a2	Ave R2	ave abs a	ave a2	Ave R2
pr1-1	Decile	Equal	Mktprem			0.4263	0.2983	0.69			
pr1-1	Decile	Equal	Mktprem	SMB	HML	0.2816	0.2229	0.87			
pr12-2	Decile	Equal	Mktprem			0.4545	0.2750	0.67	0.3370	0.1647	0.79
pr12-2	Decile	Equal	Mktprem	SMB	HML	0.2986	0.1439	0.85	0.3310	0.2097	0.90
pr60-13	Decile	Equal	Mktprem			0.4298	0.2241	0.73	0.2680	0.0899	0.80
pr60-13	Decile	Equal	Mktprem	SMB	HML	0.1478	0.0313	0.89	0.0920	0.0114	0.92

The three-factor model is clearly superior over the CAPM for all types of mean reversion and acceleration, as evidenced by the average adjusted R^2 values and average intercept values. However, with intercept values far above zero for the mean continuing portfolios of the prior 12 to 2 months as well as the short-term reversal portfolios (prior 1-1), the three-factor model does not reasonably explain asset pricing. With intercept values being much closer to zero with the long-term reversal dependent portfolios, the multi-factor modeling process better explains long-term reversion to the mean. This was also found in Fama and French (1996). Note the similarity of results between the more inclusive period of 522 months and the 1963-1993 period of 366 months. The low average alpha values of the prior return portfolios also close to the average alpha estimates for dependent portfolios formed on size and value to price (2008:1).

The drop in explanatory power and intercept values for the 522 month period versus the 366 month regressions was also seen in 2008:1, where dependent portfolios of size and various accounting ratios were analyzed. The decrease may stem from greater uncertainty being introduced into the regression runs from the return data. As suggested in 2008:1, the greater variability may be due to either the 2000 era tech bubble now being in the data, or changes occurring at the macro level, or both. It will be interesting to see if explanatory power of the more inclusive period increases when macro variables are added to the modeling process. Another explanation is that size and value premiums have simply declined in recent years. This is a large finding of Cochrane (1999). The results of this paper are consistent with such a decline in size and value premiums. The declines could be due to wider knowledge and usage of strategies based on these factors.

Overall, long-term reversion of assets can be explained by the above regressions, but the multi-factor model has a more difficult time with short-term reversal and momentum.

3. A Fourth Factor

Methodology / Data. Carhart (1997) is regularly cited in the literature for finding that almost all mutual funds revert to the mean within five years of initial year performance measurement. While being an important conclusion in itself, the real value of citing Carhart in this working paper is due to Carhart's introduction of a fourth independent variable into the Fama and French factoring process.

Adding a fourth factor for momentum is intended to more directly capture the momentum effect found by Jegadeesh and Titman's (1993). A four-factor model can be seen as generating coefficients and premia that are attributable to four different strategies of size, value, high vs low beta, and momentum vs contrarian. See, Carhart (1997), at 61. Carhart found that adding a momentum independent variable substantially improved the modeling output, reducing the average pricing errors of both the CAPM and the Fama and French three-factor model. The econometric model becomes:

$$R_{it,\,t-1} - R_{ft,\,t-1} = \alpha_i + b_{1i}\,(R_{m,\,t-1} - R_{ft,\,t-1}) + b_{2i}\,SMB_{t-1} + b_{3i}\,MOM_{t-1} + \varepsilon_{it,\,t-1} \qquad 2)$$

The model is now more accurately stated as a time series equation. The momentum factor, MOM, is formed by calculating the average return on two high prior 12 to 2 month return portfolios minus the average return on the two low prior return portfolios. Data on the momentum variable is obtained from the Kenneth French on-line data library. R_{it} is formed by equally-weighted portfolios being constructed monthly using prior 12 to 2 month return decile breakpoints (Pr12dec1-10).

Carhart only developed a momentum variable to model short-term mean acceleration. The French data library also contains reversionary factors, so those items are modeled in this paper as well, using the same general procedure as with the momentum factor. For short-term reversal of a single month, the dependent asset portfolios are constructed monthly using prior one-month return decile breakpoints (Pr1dec1-10). A short-term reversal factor (ST_Rev) is used as the fourth factor, which is the average return on the two low prior month return portfolios minus the average return on the two high prior return portfolios. For long-term reversal, the dependent asset portfolios are constructed monthly using prior 60 to 13 month return decile breakpoints (Pr60dec1-10). A long-term reversal factor (LT_Rev) is now used for the fourth factor, being the average return on the two low prior 60 to 13 month return portfolios minus the average return on the two high prior return portfolios.

The same dependent portfolios are used in the four-factor model as with the Fama and French model in Part II. The only difference in the two models is the insertion of the additional independent factor to measure the effect that momentum, short-term reversal, or long-term reversal has upon asset pricing.

Estimation Results. The following summary statistics are reported for the fourth independent factor. Summary information on the prior return dependent portfolios is noted in Part II, above.

Table VIII – Summary Statistics, Independent Variables

Summary Statistics

Variable	N	Mean	StdDev	t Value	Pr > |t|
Mom	522	0.8175	4.0001	4.6700	<.0001
ST_Rev	522	0.6176	3.0682	4.6000	<.0001
LT_Rev	522	0.3595	2.4613	3.3400	0.0009

Correlations

	MktPrem	SMB	HML	Mom
MktPrem	1.0000			
SMB	0.2747	1.0000		
HML	-0.3914	-0.2790	1.0000	
Mom	-0.0621	-0.1538	0.0103	1.0000

	MktPrem	SMB	HML	ST_Rev
MktPrem	1.0000			
SMB	0.2747	1.0000		
HML	-0.3914	-0.2790	1.0000	
ST_Rev	0.2479	0.2024	-0.0942	1.0000

	MktPrem	SMB	HML	LT_Rev
MktPrem	1.0000			
SMB	0.2747	1.0000		
HML	-0.3914	-0.2790	1.0000	
LT_Rev	-0.0956	0.1602	0.4504	1.0000

The portfolios formed with mean acceleration or momentum exhibit a substantial improvement in the alpha intercept, so much so that the model now is in the same range as the size and book portfolio sorts with the three-factor model.

Table IX - 1963-2006, Prior 12 - 2 month Dependent Portfolios (MOM)

$$R_i - R_f = a + b\ (R_m - R_f) + s\ (SML) + h\ (HML) + m\ (MOM) + u_i$$

	Pr1-1	Pr1-2	Pr1-3	Pr1-4	Pr1-5	Pr1-6	Pr1-7	Pr1-8	Pr1-9	Pr1-10
a	0.1003	0.1349	0.1562	0.1376	0.1663	0.1155	0.1844	0.1867	0.2729	0.3459
t(a)	0.42	1.11	1.76	1.80	2.27	1.70	2.57	2.30	2.76	2.61
b	1.1493	0.9892	0.9424	0.9095	0.9019	0.9018	0.9250	0.9442	1.0022	1.1692
t(b)	23.24	40.91	53.23	58.56	62.23	66.43	68.38	68.89	60.06	43.96
s	1.1143	0.8213	0.7364	0.6482	0.6486	0.6274	0.6168	0.6299	0.6847	0.8131
t(s)	17.0700	25.49	31.22	31.38	33.57	34.70	34.06	34.04	30.39	22.97
h	0.2612	0.3627	0.4190	0.3949	0.4213	0.4108	0.4121	0.3733	0.3671	0.2213
t(h)	3.42	9.59	15.14	16.31	18.57	19.36	19.33	17.04	13.77	5.33
m	-0.6684	-0.3914	-0.2846	-0.1690	-0.7983	0.0042	0.0766	0.1815	0.2929	0.5273
t(m)	-13.57	-16.19	-16.08	-10.89	-5.51	0.31	5.67	13.27	17.59	19.83
adj. R2	0.73	0.87	0.91	0.92	0.92	0.93	0.93	0.93	0.91	0.87
F test	354.55	843.41	1287.97	1414.20	1530.03	1683.70	1711.56	1728.41	1360.25	839.18

The portfolios formed with short-term reversion also exhibit improvement in explanatory power as well as having an intercept closer to zero than wit the three-factor model. Note that all independent variables continue to be individually and jointly significant.

Table X - 1963-2006, Prior 1 - 1 month Dependent Portfolios (ST Rev)

$$R_i - R_f = a + b\,(R_m - R_f) + s\,(SML) + h\,(HML) + st\,(ST\ Rev) + u_i$$

	Pr1-1	Pr1-2	Pr1-3	Pr1-4	Pr1-5	Pr1-6	Pr1-7	Pr1-8	Pr1-9	Pr1-10
a	0.1003	0.1349	0.1562	0.1376	0.1663	0.1155	0.1844	0.1867	0.2729	0.3459
t(a)	0.42	1.11	1.76	1.80	2.27	1.70	2.57	2.30	2.76	2.61
b	1.1493	0.9892	0.9424	0.9095	0.9019	0.9018	0.9250	0.9442	1.0022	1.1692
t(b)	23.24	40.91	53.23	58.56	62.23	66.43	68.38	68.89	60.06	43.96
s	1.1143	0.8213	0.7364	0.6482	0.6486	0.6274	0.6168	0.6299	0.6847	0.8131
t(s)	17.0700	25.49	31.22	31.38	33.57	34.70	34.06	34.04	30.39	22.97
h	0.2612	0.3627	0.4190	0.3949	0.4213	0.4108	0.4121	0.3733	0.3671	0.2213
t(h)	3.42	9.59	15.14	16.31	18.57	19.36	19.33	17.04	13.77	5.33
m	-0.6684	-0.3914	-0.2846	-0.1690	-0.7983	0.0042	0.0766	0.1815	0.2929	0.5273
t(m)	-13.57	-16.19	-16.08	-10.89	-5.51	0.31	5.67	13.27	17.59	19.83
adj. R2	0.73	0.87	0.91	0.92	0.92	0.93	0.93	0.93	0.91	0.87
F test	354.55	843.41	1287.97	1414.20	1530.03	1683.70	1711.56	1728.41	1360.25	839.18

The long-term reversal factor also brings small enhancements to the explanatory power of asset pricing, even though results were within tolerable levels with the three-factor model.

Table XI - 1963-2006, Prior 60 –13 month Dependent Portfolios (LT Rev)

$$R_i - R_f = a + b\,(R_m - R_f) + s\,(SML) + h\,(HML) + lt\,(LT\ Rev) + u_i$$

	Pr60-1	Pr60-2	Pr60-3	Pr60-4	Pr60-5	Pr60-6	Pr60-7	Pr60-8	Pr60-9	Pr60-10
a	0.3561	0.1100	0.0393	0.0664	0.1525	0.1559	0.1204	0.1113	0.0761	-0.1823
t(a)	1.91	1.17	0.51	0.95	2.25	2.22	1.89	1.66	0.92	-1.75
b	1.1230	1.0279	0.9876	0.9561	0.9288	0.9232	0.9445	0.9628	1.0338	1.1691
t(b)	25.91	47.59	55.73	31.98	68.60	71.51	69.56	73.61	65.64	54.32
s	1.0709	0.7465	0.6166	0.6280	0.5425	0.5440	0.5309	0.5671	0.5761	0.6590
t(s)	18.06	25.24	25.41	31.98	29.05	30.45	28.43	31.40	26.47	22.24
h	0.4030	0.3464	0.3857	0.4200	0.4088	0.4012	0.4279	0.3870	0.3029	0.1275
t(h)	5.24	9.02	12.24	16.20	16.53	16.82	17.45	16.14	10.45	3.26
lt	0.7351	0.4399	0.3155	0.1213	0.0345	-0.0417	-0.1339	-0.2408	-0.2723	-0.3813
t(lt)	8.80	10.55	9.22	4.34	1.29	-1.63	-5.05	-9.33	-0.88	-9.05
adj. R2	0.75	0.89	0.91	0.93	0.93	0.93	0.93	0.94	0.92	0.90
F test	382.61	1018.35	1246.95	1716.31	1671.80	1785.90	1684.13	1917.40	1547.44	1183.57

Comparing the single-factor market model, the three-factor model of Fama and French, and the four-factor models is done in the following summary table. Notice the continuing improvement in explanatory power and reduction in alpha intercept values as additional independent variables are modeled.

Table XII - Summary Results, All Models

							KCK 63-06			FF 63-93		
Depend.	Sort	Weight	Explan.	Explan.	Explan.	Explan.	ave abs a	ave a2	ave R2	ave abs a	ave a2	ave R2
pr1-1	Decile	Equal	Mktprem				0.4263	0.2983	0.69			
pr1-1	Decile	Equal	Mktprem	SMB	HML		0.2816	0.2229	0.87			
pr1-1	Decile	Equal	Mktprem	SMB	HML	ST-Rev	0.1781	0.1046	0.89			
pr12-2	Decile	Equal	Mktprem				0.4545	0.2750	0.67	0.3370	0.1647	0.79
pr12-2	Decile	Equal	Mktprem	SMB	HML		0.2986	0.1439	0.85	0.3310	0.2097	0.90
pr12-2	Decile	Equal	Mktprem	SMB	HML	Mom	0.1801	0.0376	0.89			
pr60-13	Decile	Equal	Mktprem				0.4298	0.2241	0.73	0.2680	0.0899	0.80
pr60-13	Decile	Equal	Mktprem	SMB	HML		0.1478	0.0313	0.89	0.2680	0.0114	0.92
pr60-13	Decile	Equal	Mktprem	SMB	HML	LT-Rev	0.1370	0.0258	0.90			

Conclusion

Findings. Using prior return dependent portfolios with the three-factor model produces greater explanatory power as to both momentum and reversionary tendencies in asset pricing than with a single-factor market model. Inferential testing of the independent variables also demonstrates that the beta coefficients are individually and jointly significant. The alpha intercept is close to zero with prior return portfolios based on mean reversion, indicating that the three-factor model adequately explains reversion of equities from a 12 month to 60 month period prior to portfolio formation, at least for the time period tested (522 months). However, the hypothesis test that alpha is zero is rejected with mean acceleration over the first 12 months prior to portfolio formation, as well as with very short-term reversals. Both the results of this study and those of Fama and French (1996) have intercepts with returns in excess of risk premiums for the independent variables.

The addition of a fourth variable that more directly measures the momentum and mean reversion of asset pricing improves results. The momentum portfolios are considerably closer to zero on the intercept and have higher adjusted R^2 values. Marginal improvements also occur with the reversionary portfolios in explanatory power.

Future Research. The items that should be addressed in developing a multi-factor model that supplements the single-factor CAPM include:

- Developing macro factors as additional independent variables.
- Possibly adding a time trend.
- Adding the GRS (1989) F test for portfolio efficiency.
- Stress testing the variables at the extremes.
- Stating the model in terms of returns attributable to each of the independent variables.
- Exploring underlying or "state" variables for the independent factors. A fundamental analysis of asset pricing over successively longer time periods may reveal a far different picture of the explanatory factors to asset pricing. Indeed, several papers have shown an increasingly strong relationship between dividend yield or earnings growth and asset returns over long time frames. See, Hagstrom (1999; Appendix B); Cochrane (1991); Fama and French (1988; 1989). Long-term investors may simply view the investment universe in terms of highly predictable cash flows over long stretches of time, for instance. For such investors, asset return may be a basic matter of growth in free cash flows of a business, rather than an overly theoretical view of market, size, and value risk premiums.
- Perhaps most importantly, stating the returns of the model on a post-tax, post-cost basis. Numerous studies indicate that active styles that seek to capture size, value,

momentum, etc, under-perform the relevant benchmark indices once tax impacts and costs of research are accounted for. In fact, even though Carhart (1997) pioneered an additional variable in the multi-factor model for momentum, he concluded that the high trading activity of momentum tactics is not economically profitable after costs are accounted for. Additionally, since the vast bulk of actively managed funds under-perform the market indices, the implication is that investments professionals cannot on the average use size, value, momentum, and reversionary strategies to overcome their own costs of research and tax impacts. Even for the funds that do out-perform, Carhart (1997) also found that almost all of the persistence in return is due to persistence in the underlying stocks, and not due to manager skills.

References

Nicholas Barberis, "Investing in the Long Run When Returns are Predictable", *The Journal of Finance* no. 1 (2000): 225-265.

F. Black, F., M. C. Jensen, and M. S. Scholes, The Capital Asset Pricing Model: Some Empirical Test, in Michael C. Jensen (ed.) Studies in the Theory of Capital Markets. New York: Praeger. (1972): 79–121.

Zvi Bodie, Alex Kane, and Alan J. Marcus, *Essentials of Investments,* McGraw-Hill Irwin, 2004.

Mark Carhart, "On Persistence in Mutual Fund Performance", Journal of Finance 52 (1997): 57-82.

Louis K.C. Chan & Josef Lakonishok, "Value and Growth Investing: Review and Update", Financial Analysts Journal, Jan / Feb. 2004, pp. 71-84.

Navin Chopra, Josef Lakonishok, Jay R. Ritter, "Measuring Abnormal Performance: Do Stocks Overreact?" *Journal of Financial Economics* 31 (1992): 235-68.

John H. Cochrane, “Volatility Tests and Efficient Markets: Review Essay”, *Journal of Monetary Economics,* Vol. 27, No. 3, June 1991: 463-485.

Aswath Damodaran, Investment Valuation, John Wiley & Sons, 1996.

Walter F.M. DeBont and Richard H. Thaler, “Does the Stock Market Overreact?”, *Journal of Finance,* July, 1985.

Walter F.M. DeBont and Richard H. Thaler, “Further Evidence on Investor Overreaction and Stock Market Seasonality”, *Journal of Finance,* July, 1987.

Eugene F. Fama and Kenneth R. French, "Migration", CRSP Working Paper No. 614, Feb. 2007.

Eugene F. Fama and Kenneth R. French, “Multifactor Explanations of Asset Pricing Anomalies”, *Journal of Finance* 51, no. 1 (March 1996): 55-84.

Eugene F. Fama and Kenneth R. French, “Size and Book-to –Market Factors in Earnings and Returns”, *Journal of Finance* 50, no. 1 (March 1995): 131-155.

Eugene F. Fama and Kenneth R. French, “Common Risks Factors in the Return of Stocks and Bonds”, *Journal of Financial Economics* 33, no. 1 (February 1993): 3-56.

Eugene F. Fama and Kenneth R. French, “The Cross Section of Expected Stock Returns”, *Journal of Finance* 47 (1992): 427-466.

Eugene F. Fama and Kenneth R. French, “Business Conditions and Expected Returns on Stocks and Bonds”, *Journal of Financial Economics*, Vol. 25, No. 1, November, 1989: 23-49.

Eugene F. Fama and Kenneth R. French, “Dividend Yields and Expected Stock Returns,” *Journal of Financial Economics* 22 (Oct. 1988): 3-25.

Eugene F. Fama, “Efficient Capital Markets: A Review of Theory and Empirical Work”, *Journal of Finance* 25 (1970): 383-417.

Robert Hagstrom, *The Warren Buffett Portfolio*, 1999.

Narasimham Jegadeesh and Sheridan Titman, “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency”, *Journal of Finance* 48 (March, 1993): 65-91.

Kevin C. Kaufhold, “Investigating Price to Value Indicators in the Capital Equity Markets”, working paper 2008:1.

James M. Poterba and Lawrence H. Summers, "Mean Reversion in Stock Prices, Evidence and Implications", *Journal of Financial Economics* 22 (Oct. 1988): 27-59.

James O'Shaughnessy, *What Works on Wall Street* (1998).

Reichenstein & Dorsett, "Time Diversification Revisited", AIMR publication, 1995: 46-47.

Jeremy Siegel, *Stocks for the Long Run,* Irwin, 2^{nd} ed, 1998.