# On Capital Deepening

Kevin C. Kaufhold

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## *1. Introduction*

The capital-labor ratio is analyzed in this paper, with an emphasis on the chronological development of the economic literature.

The possibility that capital is comprised of heterogeneous components is explored in Section 2. The neoclassical model on capital formation is developed in Section 3, with the focus being on the model's theoretical underpinnings. Various extensions to the basic model are then examined in Section 4. This discussion points towards the productivity growth arena, and Section 5 analyzes the capital to labor ratio in terms of overall productivity. The capital to labor ratio is often viewed as being a critical part of labor productivity. The decomposition of labor productivity is thus examined in Section 6. Lastly, Section 7 summarizes the various ways in which the capital deepening process is currently viewed.

## *2. Heterogeneity of Capital*

Much of the discussion regarding capital inputs, including vintage capital and capital deepening, starts with analysis provided by Solow (1960). According to Solow, the homogenous nature of capital accumulation and replacement appears to be unrealistic, at least from the perspective of the individual firm. Solow suggested accounting for the age or vintage of the capital. With this suggestion, vintage capital became an important part of the growth literature. Solow describes vintage technology in terms of the Cobb-Douglas function:

$$T(v, t) = [\, e^{\gamma v} I(v) \,]^{1-\alpha} L(v, t)^{\alpha} \tag{1}$$

Where, I is investment, and L is labor. Quality adjustments occur to capital at the rate $\gamma$. Quality adjusted capital becomes:

$$K(t) = \int_{-\infty}^{t} e^{\gamma v} I(v)\, dv \qquad (2)$$

Technological progress is expressed as a steady improvement in equipment quality:

$$A'(t) = I(t) - \gamma A(t) \qquad (3)$$

Capital obsolescence appears in the Solow model as a declining value of capital, and ultimately, as a comparative reallocation to labor. As noted by Solow, this can serve to explain various productivity slowdowns. A declining capital to labor ratio may thus be the result of an aging capital base. For instance, Baily (1981) traced major structural changes in the economy to the obsolescence of existing capital stock.

Some papers began to view quality changes of assets of different ages and the summation of heterogeneous component parts of the capital stock in terms of potential measurement biases and errors.[1] By the early 1970's, the quality of capital as an explanatory factor in the capital process generally receded as a theoretical topic of discussion. This may have been largely due by the realities of the times: most of the WWII era productive capacity of the nation had simply been replaced by the 1970's. Additionally, as innovations in technology occurred and a "technology revolution" commenced, the focus of the literature shifted towards Information Technology (IT) related issues. The aging of the capital base was not so much an issue as was a more complete theoretical understanding for the variability of capital and labor inputs. Research into vintage capital gradually evolved into an exploration of the capital deepening process.

[1] See for example, Bosworth (1982) for a nice examination of this issue.

## *3. The Theoretical Basis of Capital Deepening*

Burmeister and Turnovsky (1972) set forth the basic concepts of the capital deepening process. In a one sector model, an increase in the capital-labor ratio occurs in response to a decrease in the interest rate. Starting with a production function of $Y = F(K, L)$, where K and L are the capital and labor inputs, the per capita form becomes $y = f(k)$, where $y = Y / L$; $k = K / L$; $f'(k) > 0$, and $f''(k) < 0$. Assuming depreciation occurs at rate $\delta$, and g is the rate of growth in labor, the steady state per capita consumption becomes:

$$c = f(k) - (g + \delta) k \tag{4}$$

Where c is per capita consumption. This flows from the national income identity of $Y = C + I$, with I now being gross investment. Per capita consumption also can be seen simply $c = C / L$. The steady state capital-labor ratio is:

$$f'(k) = r + \delta \tag{5}$$

Where, r is the rate of interest which is determined exogenously. With $f''(k) < 0$, k can be solved in terms of r. Capital deepening essentially now turns on k's response to r. With $dk / dr = 1 / f''(k) < 0$, a decrease in the interest rate will unequivocally cause an increase in the steady-state capital-labor ratio k. In a one-sector model, per capita consumption, c, will also have a unique maximum at $r = g$. Therefore, the capital-labor ratio in a one sector model will always vary inversely with regard to the interest rate while consumption is considered well-behaved, having a maximum at the "Golden Rule" of $r = g$.

When heterogeneous capital goods exist, the capital deepening process may be ambiguous, however. Starting with per capita capital of k and per capita output of y, consumption, c, can be generally stated as a a production possibility frontier (PPF) of $c = T(y_1, \ldots, y_n; k_1, \ldots, k_n)$. At steady-state, $y_i = (g + \delta_i) k_i$, the PPF becomes:

$$c = T [(g + \delta_1) k_1,\ldots, (g + \delta_n) k_n; k_1,\ldots, k_n] \quad (6)$$

Thus, the equilibrium is now expressed in terms of the labor growth rate, capital depreciation, and per capita capital. The analysis can now be expanded to a multi-sector framework. Within each sector, capital deepening may not be completely responsive to interest rate changes. Even if k is "regular" in the sense of the unequivocal result of a single-sector model, heterogeneous modeling may not be regular in the aggregate. This depends on the capital intensiveness of different sectors and how labor shifts between them in response to interest rate changes. If capital deepening responses are sufficiently strong, the aggregate economy is likely to be regular and close to $r = g$.

## *4. Extensions of the Basic Model*

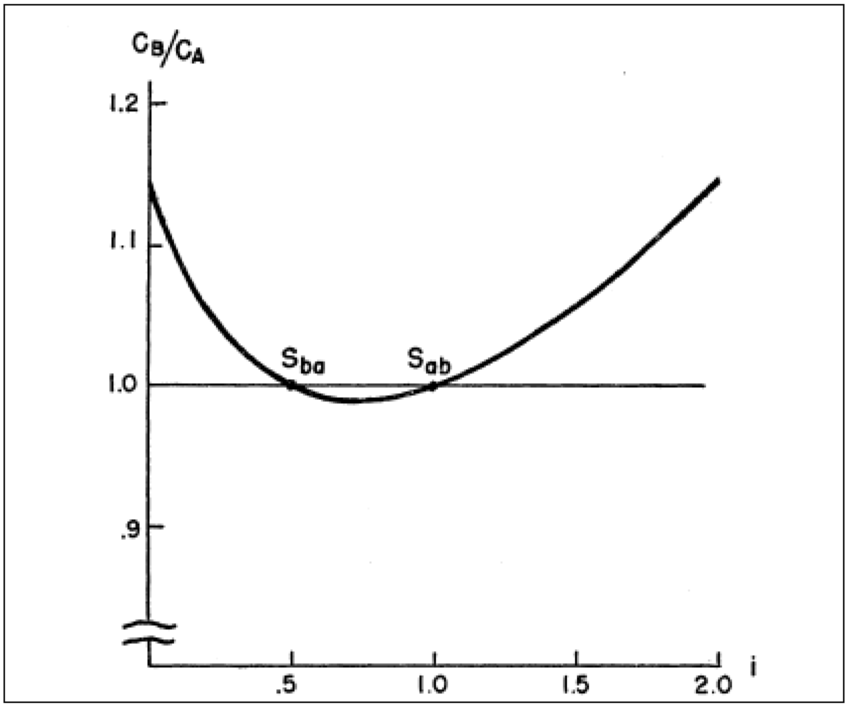
One of the theoretical studies that continued to examine heterogeneous capital was Hirota (1979). A "non-paradoxical" consumption behavior results when a long run balance is imposed such that a constant k is maintained with a constant rate of population growth, g. The per capita consumption that is sustainable is: $c = f(k) - (g + \delta) k$. The consumption now occurs within the context of an aggregate neo-classical model. Citing to Burmeister and Turnovsky (1972), Hirota agreed that a steady state economy with many capital goods exhibits non-paradoxical consumption behavior if the economy exhibits a capital deepening response for each local economy's own rate of interest and in the range of the rate of population growth, $r \approx g$. Hirota goes on to demonstrate that capital deepening occurs if the fall in the own interest rates produces an increase in the value of per worker capital good stock (this is referred to as the "Cambridge" type of response):

$$d\left(\sum^{n}_{i=1} p_i(r) * k_i(r)\right) / dr < 0 \tag{7}$$

Also noted is the earlier capital deepening response of Burmeister and Turnovsky (1972), wherein: $\sum^{n}_{i=1} p_i(r) * d\, k_i(r)) / dr < 0$. Hirota proposes that an economy on an optimal growth path is paradoxical and not regular around a steady state if and only if it has an unstable saddle point property. Thus, the issue of "regularity" in the sense of the literature may be related to the problem of a heterogeneous capital goods economy.

Another paper explored the possibility of a paradoxical result occurring from a change in interest rates. Samuelson (1966) showed that a decrease in interest rates would decrease the costs of production, and ultimately increase output and capital spending in order to produce that output, only if the cost optimization solution was one-directional in nature. Costs functions can generate a bi-modal solution. This is shown in Figure 1 (from Samuelson, 1968):

**Figure 1 - Bi-Modal Solution to the Costs of Production**




The production cost function can produce two possible equilibriums. Samuelson referred to this as “re-switching”. A decrease in interest rates could provoke either an increase in output (which would be the regular response expected by neoclassical theory), or a decrease in output. Far from being unequivocal, the exact output selected by a firm would actually be indeterminate. Samuelson’s bimodal solution becomes almost prophetic when one adds into the analysis current-day discussions on consumer and investor confidence. If consumer demand is weak, producers may very well choose the lower equilibrium output level, rather than the higher level, in response to a lower interest rate environment.

Not only were various papers showing the theoretical possibility of non-regular responses in the capital structure of the economy, by the late 1970’s aggregate data was also conflicting with the neoclassical models. This was demonstrated in Bosworth (1982), who showed that the capital-labor ratio dramatically declined from 3.3 in 1951-1955 to 0.6 by 1976-1980. At the same time however, the ratio of capital stock to gross output increased from 1.26 in the 1950's to 1.43 by 1976-1980. The decrease in capital deepening may have been consistent with higher inflation and restrictive monetary policy in the 1970's. But the loss of capital deepening started far before the tighter monetary regime of the 1970's, back to a time of a vast expansionary fiscal policy in the 1960's. Additionally, the continual increases in the capital to output ratio was inconsistent with restrictive macro policies of the mid to late 1970's. Further highlighting the problem was that nonresidential investment as a percent of gross and net output continued to increase for the entire period of review, from the 1950's to the late 1970's.

Bosworth examined several potential explanatory factors, including measurement error of capital and investment ratios, the premature obsolescence and aging of capital, capital taxation issues, interest rates, and inflationary issues. Bosworth concluded that investment "held up surprisingly well" because higher inflationary levels effectively reduced the real cost of capital. With monetary policy becoming more restrictive by the end of the 1970's, Bosworth felt the result might be substantial weakening of investment into the 1980's, with capital "rotting from lack of use". Commentary on the paper, most notably from Robert Solow and Laurence Summers, felt that while numerous issues were thoughtfully developed, more questions were raised than answered by the effort (Bosworth, 1981).

## *5. Decomposing Labor Productivity*

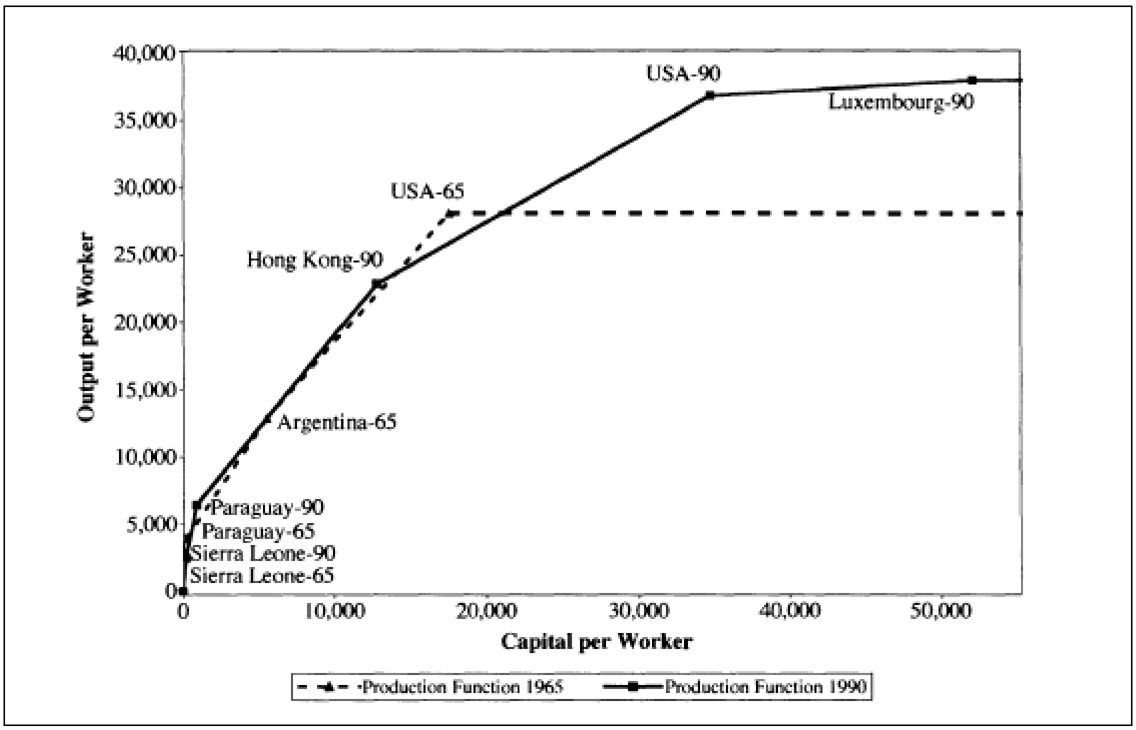
Partly as a response to continuing riddles regarding conflicting macro-economic data on capital and investment, the literature by the 1980's was analyzing capital and labor inputs in much greater detail. The search was on for the factors causing the inputs themselves to vary. Regression analysis was initially attempted to sort through the multiple variables involved in both capital formation and labor productivity. But several commentators, including Quah (1993), argued that standard regression models focusing on the first moments of a probability distribution had difficulties addressing convergence issues, especially if incomplete or partial historical data was utilized. The focus turned to Growth Accounting procedures, with labor productivity being decomposed into component parts. Research generally concentrated on the capital to labor ratio; changes in education and skill levels; and total factor productivity in explaining the variability of labor productivity. Capital deepening issues thus became subsumed within a broader analysis of labor and technological productivity.

A good example of this process is contained in Kumar and Russell (2002) who decomposed the factors affecting labor productivity. A world-wide production function was developed, with decomposition of factors affecting labor productivity then being done for 57 countries between 1965 and 1990. Constant returns to scale was assumed, with $k = K / L$ and $y = Y / L$, across base period b and current period c. Change of efficiency, $e_c / e_b$, is measured as the change in distance from the world-wide production frontier; the technology changes of $y_c (k_b) / y_b (k_b)]$ will shift the frontier; and the effect of a change in the capital-labor ratio, $y_c (k_c) / y_c (k_b)]$, is shown as movement along the frontier. The decomposed equation is:

$$y_c / y_b = [e_c / e_b] * [y_c (k_b) / y_b (k_b)] * [y_c (k_c) / y_c (k_b)] \quad (8)$$

With this equation, the change in output across two time periods is now expressed as the change to the component parts of efficiency, technology, and the capital-labor ratio. The world-wide production frontier at the beginning of the sample data in 1965 and the end of the data in 1990 can be visually portrayed, as shown in Figure 2 (from Kumar and Russell, 2002):

**Figure 2 - World-Wide Production Frontier**




Notice the vast expansion of the production frontier among the highly industrialized nations, while countries along the lower end of the frontier show almost no change. Kumar and Russell describe this as a "decidedly non-neutral" in technological change, with technology largely benefiting the nations with high capital-labor ratios. In fact, technological change produced an "implosion" of the production frontier at very low capital-labor ratios (for example, Paraguay, Sierra Leone); modest expansion at relatively low capital-labor ratios, and rapid economic expansion at high capital-labor ratios (for example, USA, Luxembourg).

Technology assisting the higher capital-labor nations more so than the countries with lower capital-labor ratios is more directly shown in the following graph (from Kumar and Russell, at 540):

**Figure 3 - Capital-Labor Ratio, 1965 and 1990**

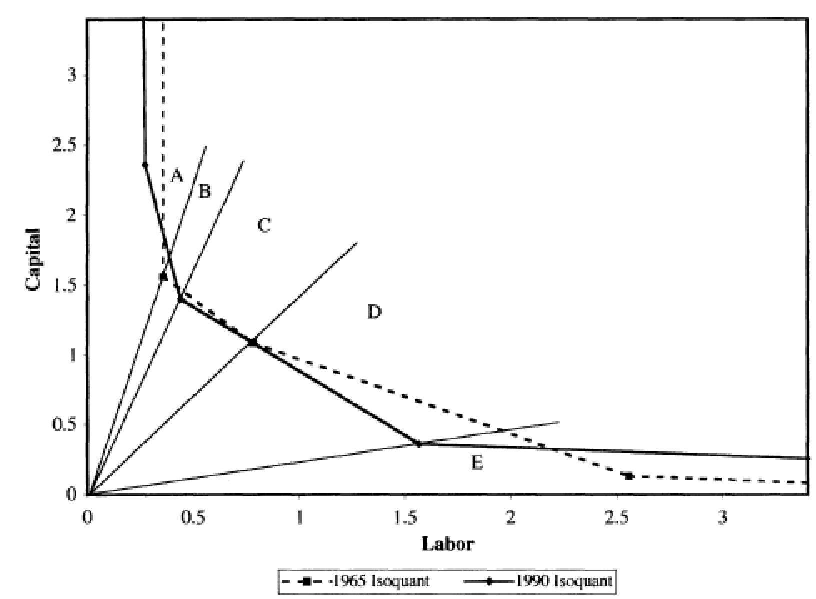


FIGURE 8. 1965 AND 1990 ISOQUANTS

The 1965 and 1990 isoquants are shown in the graph as dashed and solid curves, respectively. Notice that the nations starting off with high capital-labor ratios in 1965 (upper left of the graph) are pushed inward, to even greater capital deepening by 1990. Conversely, the economies beginning at low capital-labor levels in 1965 (lower right of the graph) retrograde by 1990, requiring more capital for the same labor. The more productive nations could produce the same output by 1990 with some 25% less capital and labor inputs, whereas the nations with low capital deepening levels actually required 30% factor inputs for the same output. (Kumar and Russell, at 540).

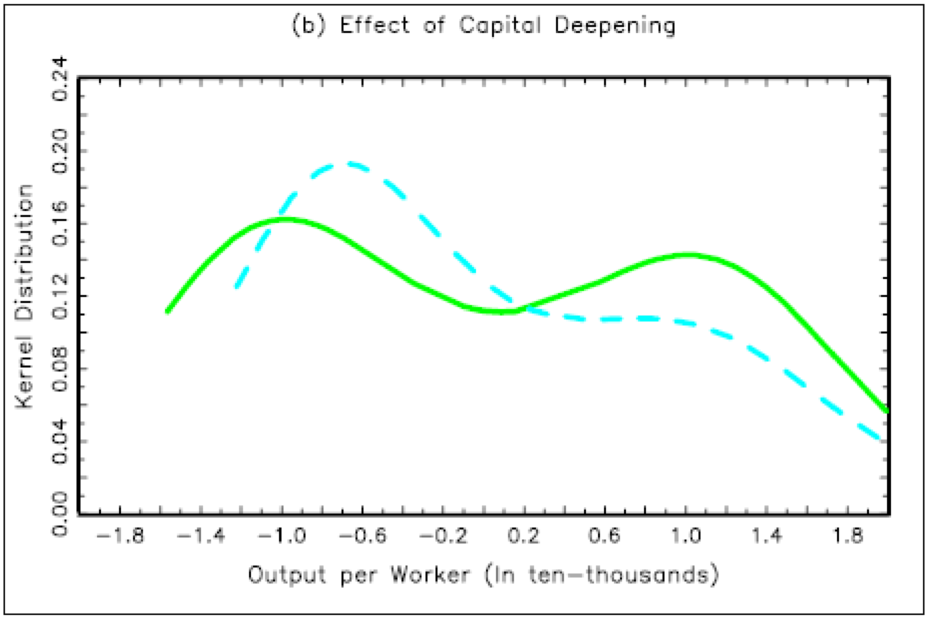
A similar result was shown in Henderson and Russell (2005). Decomposition among 52 countries was done for the same period, 1965-1990. A growth accounting format was utilized, with Cobb-Douglas initially being specified, $Y_t = a_t K_t^{1/3} (H_t L_t)^{2/3}$, where Y, K, L are aggregate output, physical capital, and human capital, respectively; and $a_t$ is the Solow residual. The tri-parte decomposition becomes:

$$y_c / y_b = (a_c / a_b) * (k_c / k_b)^{1/3} * (H_c / H_b)^{2/3} \tag{9}$$

With the first term, $(a_c / a_b)$, being the Solow residual or the contribution of productivity growth of technological change; the second part being capital deepening; and the third part of the equation measuring the contribution of human capital accumulation. The Penn World Data Table was used as the data sources of output, capital, and labor. A two-club international polarization was generated primarily by technological change as evidenced by changes in the Solow residual. The increased dispersion of the distribution was largely due to capital deepening, with some assistance from technological change and human capital accumulation.

Perhaps the most interesting aspect of both Kumar and Russell, and Henderson and Russell is the visual presentation of the two-club convergence. The following graph is portrayed as the effect from capital deepening, but similar graphs also show the effect as from human capital accumulation and changes in the Solow residual (from, Henderson and Russell, at 26):

**Figure 4 - Capital Deepening Effect on Divergence**




The bipolar divergence of the productivity distribution is clearly demonstrated in Figure 3. The solid line is 1990 data, while the dashed line of Figure 3 represents 1965 data. The mean average is the represented by the vertical dashed lines. The authors found capital deepening to be the driving force behind this divergence of the distribution.

An interesting Report (National Council of Advisors, 2001) stated the decomposition as:

$$\Delta Y / Y_t = \alpha \Delta K / K_t + (1 + \alpha) \Delta L / L_t + \Delta A / A_t \tag{10}$$

The capital, labor, and technology components are identified in this tri-parte equation with the typical notation of K, L, and A. Using US data from 1948 to 2000, the following results were obtained:

**Table I – Growth Accounting Results**

Growth Accounting for the United States, 1948-2000

| | Annual Growth Rate of Y | Annual Growth Rate of Y/L | Contribution of K/L | Annual Growth Rate of A |
|---|---|---|---|---|
| 1948-1973 | 4.0% | 3.0% | 1.2% | 1.8% |
| 1973-1995 | 2.7% | 0.9% | 0.8% | 0.1% |
| 1995-2000 | 4.2% | 3.0% | 1.1% | 1.9% |

($\alpha = 0.4$)

The Report attributed the drop in capital deepening in the middle period to the oil shocks of the 1970's, and the large Reagan budget deficits of the 1980's. The post 1995 acceleration of capital deepening results from the end of budget deficits; and the rapid fall in price of information types of capital. Changes in TFP were thought to be most significant of all three tested factors, however. The "information technology revolution" was the driving force on this substantial improvement, according to the Report.

Also decomposing productivity growth, Jorgenson, et al (2002) found that the increase in output growth after 1995 was due to more rapid capital deepening as well as faster non-IT types of TFP growth. The increase in capital deepening was because the growth of hours worked was less than the growth in non-IT capital, resulting in an increase in non-IT capital per hour worked.

Steindel and Stiroh (2001) examined the data compilation in great detail. Finding that gains primarily reflected more rapid capital deepening and faster TFP growth, capital deepening also was believed to have a normal cyclical component. This is due to investment being highly cyclical. Results of three different studies are summarized as follows:

**Table II – Labor Productivity and Economic Growth, 1973-1999**

**Table 2: Sources of Economic Growth, 1973-99**

| Variable | BLS 1973-95 | BLS 1995-98 | BLS Change | Jorgenson and Stiroh 1973-95 | Jorgenson and Stiroh 1995-99 | Jorgenson and Stiroh Change | Oliner and Sichel 1973-95 | Oliner and Sichel 1995-99 | Oliner and Sichel Change |
|---|---|---|---|---|---|---|---|---|---|
| Growth of Output (Y) | 3.0 | 4.8 | 1.9 | 3.04 | 4.76 | 1.72 | 2.99 | 4.82 | 1.83 |
| Growth in Hours (H) | 1.6 | 2.5 | 0.9 | 1.62 | 2.18 | 0.57 | 1.58 | 2.25 | 0.67 |
| Growth in ALP (Y/H) | 1.4 | 2.3 | 0.9 | 1.42 | 2.58 | 1.16 | 1.41 | 2.57 | 1.16 |
| Contribution of Capital Deepening | 0.7 | 0.8 | 0.1 | 0.85 | 1.34 | 0.49 | 0.77 | 1.10 | 0.33 |
| Contribution of Labor Quality | 0.2 | 0.3 | 0.1 | 0.24 | 0.25 | 0.01 | 0.27 | 0.31 | 0.04 |
| Contribution of TFP | 0.4 | 1.3 | 0.9 | 0.34 | 0.99 | 0.65 | 0.36 | 1.16 | 0.80 |

Notes: ALP Contributions are defined in Equation (1). All values are percentages. BLS estimates for nonfarm business sector through 1998 and numbers do not add precisely due to rounding.
Sources: BLS(2000b), Jorgenson and Stiroh (2000), and Oliner and Sichel (2000).

Field (2004) inquired as to multi-factor productivity growth (MFP). Field felt that almost all of the impact on living standards will be registered as an effect of MFP if the acceleration in labor productivity growth is used. Field felt that the more meaningful measure of factor productivity was from its growth rate, and not from the rate of improvement of the rate of improvement, or second derivative of labor productivity. Measured in this way, MFP's contribution to labor productivity growth, 1995-2000, was 1.14% percent per year; capital deepening's contribution was 1.05%; and labor composition was 0.26%. This totals 2.46% per year labor productivity growth between 1995 and 2000.

Oliner and Sichel (2006) also decomposed labor productivity growth. For the 1995-2000 period, both greater TFP growth and increasing capital deepening raised labor productivity in the US compared with a prior period between 1973 and 1995. After 2000 however, capital deepening slowed, and this was attributed to fairly weak aggregate investment. TFP types of growth in labor productivity continued to grow, resulting in an overall increase in labor productivity. This total factor productivity growth was traced to continued gains in information technology. Previously, Oliner and Sichel (2000) estimated that capital deepening from IT related components accounted for one-half of the 1% increase in labor productivity growth in the late 1990's.

## *6. Current Themes*

Capital deepening concepts are currently being used in a variety of settings. A review of recent literature shows that accounting methods as well as economic modeling are both useful in determining the impact upon economic processes from a changing capital to labor ratio.

Acemoglu and Guerrieri (2008) used an aggregate household model to examine supply-side reasons for non-balanced growth across various sectors. Differences in shares of capital combined with capital deepening leads to non-balanced growth because an increase in capital-labor ratios raise output in sectors with greater capital intensity. Conversely, there will be higher employment growth and slower growth in overall output in less-capital intensive industries. Over the long-term however, the capital-output ratios and the share of capital income in GDP becomes relatively constant.

Antony (2009) develops an economy consisting of two countries, one developed, and the other developing, to study the elasticity of substitution between capital and labor. A constant elasticity of substitution (CES) production function was used. The developed country is assumed to initially establish the technology standards. The two countries can contract out production between each other.

Based on empirical observations that the elasticity of substitution between capital and labor is significantly larger than one in developed nations and smaller for developing countries, an inequality of capital intensity between the two nations is implied, with capital intensity initially increasing in developed nations more than the lesser developed ones. For the situation where initial prices are lower in the developing country, Antony (2009) describes a convergence-like process. Lower elasticity of substitution between capital and labor in the developing nation will initially produce lower growth of capital intensity than in the developed nation. More goods will be demanded from the developing nation however, due to the initial assumption of lowered price in the developing nation. This induces a price increase in the lesser developed nation as well as an increase in the marginal productivity of capital. This attracts capital until at some point, the capital intensity of the two countries may become similar.

Burda (2008) looked at Eastern Germany since the time of re-unification. At the outset of unification, East Germany had limited international trade, outdated capital stock, and a goods and services output base that had been produced under non-competitive conditions. Significant capital formation has occurred since then however, but unemployment remains at almost double that of the West. Wages are still lower than in the West, but the gap has declined to 26%. Labor migration out of the country has been pronounced, and overall employment in the East shrank by 13% while increasing 7% in the West. Labor productivity in the East has increased 77% from 1991 to 2007, while per capita GDP has gone up by 40% in the same period. Overall, the vastly increasing capital resources coupled with decreasing labor has amounted to a massive structural change in the East German economy. Capital deepening has been rapid, and convergence is evident over a wide array of economic indicators.

Developing a neoclassical constant returns growth model, Burda (2008) goes on to explain the lower wages in the East, a higher return on capital, increasing productivity, and increasing output as being consistent with movement along an optimal growth path. With wages initially being much lower in the East, the model also predicts the extent of migration towards the West. With wages of the East now increasing but still being persistently lower than in the West, the model actually over-predicts the extent of migration out of the East during the past five years.

Dennis and Iscan (2009) use an accounting format as well as a macro-economic neoclassical model to explore structural changes in the US agricultural sector. Labor productivity is decomposed into three parts: income elasticity of demand for agricultural goods; productivity growth rates in sectors; and capital deepening. The authors found the low income elasticity of demand for agricultural goods accounted for much of the labor reallocation until the 1950's. In fact, 80% or more of the reallocation is attributable to the elasticity of demand. Beginning in the 1950's however, the differential sector productivity growth becomes more important, accounting for 12% of the structural change out of the agricultural sector. Capital deepening of agricultural activities slowed down the reallocation of labor away from agriculture from 1820 to 1900. However, between 1950-2000, capital deepening reversed and contributed 8% of the structural change away from agricultural labor. Capital deepening in this period was marked by higher productivity growth in the farm sector.

## *7. Conclusion*

Capital deepening may be part of the normal operation of the macroeconomic environment, so much so that an increase in the capital-labor ratio typically takes place as an economy moves along its optimal growth path. In this regard, capital deepening may be seen as a capital maturing, with successively smaller increases to the capital-labor ratio occurring as the economy closes in on its long-run equilibrium point. An increase in capital to labor input allocations is not inevitable, however. As Samuelson (1966) showed, a decrease in interest rates could actually lower the capital inputs, due to manufacturers choosing the lower equilibrium point of a bi-model solution. Also, increasing interest rates should predictably result in a decrease in capital to labor, for a capital "widening" effect.

The decomposition of labor productivity, as outlined in the recent literature, has provided more depth to the analysis. No longer are we left with unresolved issues regarding inconsistent macro investment and capital data, as was the case in the early 1980's. The capital to labor ratios could be going down while total factor productivity is simultaneously going up. This may result in an overall increase in labor productivity even in the face of declining capital deepening. Multiple effects are constantly occurring in a diverse economy. The worldwide production frontier of Kumar and Russell (2002) is a good example, with multiple factors of efficiency, capital deepening, and technological change all simultaneously impacting production. The fact that developed nations with high capital-labor ratios can take full advantage of technological shifts in the production frontier also assists in a better understanding of convergence clubs amidst great divergence of other economies.

Additionally, Solow's research on vintage capital regularly resurfaces in interesting and varied ways. Not only can heterogeneous capital stock impact productivity, but the general idea of capital quality is evident in more current analysis regarding technological shocks. Literature routinely differentiates between the kinds of technology being deployed (i.e., information technology, new equipment, updated physical plant, etc).

Further areas of research on capital deepening can and arguably should involve the role that investor confidence plays in the capital formation and deepening process, the possibility that capital deepening may be related to the business cycle (and quite possibly with time lags), and the continuing variety and quality that the capital stock has upon capital retention and productivity growth.

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